McGreggor, K., & Goel, A. K. (2012). Fractal Analogies for General Intelligence. In Proceedings of the Fifth Conference on Artificial General Intelligence (AGI-12), Oxford, UK.

Fractal Analogies for General Intelligence

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Abstract. A theory of general intelligence must account for how an intelligent agent can map percepts into actions at the level of human performance. We describe a new approach to this percept-to-action mapping. Our approach is based on four ideas: the world exhibits fractal self-similarity at multiple scales, the design of mind reflects the design of the world, similarity and analogy form the core of intelligence, and fractal representations provide a powerful technique for perceptual similarity and analogy. We divide our argument into two parts. In the first part, we describe a technique of fractal analogies and show how it gives human-level performance on an intelligence test called the Odd One Out. In the second, we describe how the fractal technique enables the percept-to-action mapping in a simple, simulated world.

1 Introduction

Russell & Norvig [28] characterize an intelligent agent as a function (f) that maps a perceptual history (P*) into an action (A). If we accept f: P* → A as a useful characterization of intelligence, it follows that a theory of general intelligence must account for how the intelligent agent maps percepts into actions. Although Russell & Norvig do not delve into it, we believe that a theory of general intelligence must also account for agent's performance at the level of human intelligence. In this paper, we present a novel approach to addressing the f: P* → A mapping at the level of human intelligence.

Our approach is based on four ideas: (1) the world exhibits fractal self-similarity at multiple scales [19]; (2) the design of mind at least in part reflects the design of the world [12]; (3) similarity and analogy form the core of intelligence [14]; and (4) fractal representations provide a powerful technique for similarity and analogy. The first three of these ideas are familiar in theories of nature and intelligence; however, it is the fourth idea which is new. We claim that analogy initiates with an act of being reminded, and that fractally representing that triggering percept as well as all prior percepts affords unprecedented similarity discovery, and thereby analogy-making.

We divide the argument in this paper into two parts. In the first part, we describe the general technique of fractal analogies and show how it gives human-level performance on an intelligence test called the Odd One Out. In the second, we describe how the same fractal technique enables the f: P* → A mapping in a simulated world, in which intelligent agents recognize one another and flock together.

2 Fractal Analogies and Novelty Detection

To deem some apprehended object as novel involves the complex interplay of at least two relationships [30-31]: the relationship between the observer and the observed, and the relationship between the observed and its context. The relationship between the observing agent and the observed object may vary depending upon some act taken by the observer. For example, if one wishes to appreciate an object at a higher level of detail, one might move closer to the object, or bring the object closer, resulting in the object occupying a larger expanse of the observer's field of view. This action modifies the resolution of the object: at differing levels of resolution, fine or coarse details may appear, which may then be taken into the consideration of the novelty of the object. The observed object also is appreciated with regard to other objects in its environment. Comparing an object with others around it may engage making inferences about different orders of relationships. We may begin at a lower order but then proceed to higher orders if needed. The context also sanctions which aspects, qualities, or attitudes of the objects are suitable for comparison.

Given the importance of perceptual novelty detection, there has been quite a bit of work on the topic. Markou & Singh [20-21] review statistical and neural network techniques for novelty detection. Neto & Nehmzow [24] illustrate the use of visual novelty detection in autonomous robots. Work on spatial novelty and oddity by Lovett, Lockwood & Forbus [18] centered on qualitative relationships in visual matrix reasoning problems. They showed that by applying traditional structure-mapping techniques [10] to qualitative representations, analogical reasoning may be used to address problems of visual oddity; however, they did not show where the representations come from [15].

Analogies in a general sense are based on similarity and repetition [14], and so we seek to employ a suitable representation, one which affords the capture of these qualities as well as sanctions reasoning over them. Fractals capture self-similarity and repetition at multiple scales [19]. Thus, we believe fractal representations to be an appropriate choice for addressing some classes of analogy problems. We model the relationship between the observer and the observed by starting with fractal representations encoded at a coarse level of resolution, and then adjusting to the right level of resolution for addressing the given problem. We model the relationship between the observed and its context by searching for similarity between simpler relationships, and then shifting its searches for similarity between higher-order relationships. In each aspect, these adjustments are made automatically by our strategy, by characterizing the ambiguity of a potential solution.

2.1 Visual Analogies and Fractal Representations

Consider the general form of a visual analogy problem as being A : B :: C : D, with the symbols being images. Some unknown transformation T can be said to transform image A into image B, and likewise, some unknown transformation T′ transforms image C into an unknown answer image D. The central analogy in such a visual problem may then be imagined as requiring that T be analogous to T′; that is, the answer will be whichever image D yields the most analogous transformation. That T and T' are analogous may be construed as meaning that T is in some fashion similar to T'.

The nature of this similarity may be determined by a number of means, many of which might associate visual or geometric features to points in a coordinate space, and compute similarity as a distance metric [29]. We adopt Tversky's interpretation of similarity as a feature-matching process, and seek to derive from each fractal representations a set of features for use in this matching process. Thus, we define the most analogous transform T′ as that which shares the largest number of matching fractal features with the original transform T.

The mathematical derivation of fractal image representation expressly depends upon the notion of real world images [2]. A key observation is that all naturally occurring images appear to have similar, repeating patterns. Another observation is that no matter how closely one examines the real world, one may find instances of similar structures and repeating patterns. These observations suggest that images may be described in terms that capture the observed similarity and repetition alone, without regard to shape or traditional graphical elements.

Computationally, determining the fractal representation of an image requires the use of the fractal encoding algorithm. We refer the interested reader to our earlier work for the details of this algorithm [16, 22].

Spatial		***Photometric***	
s_x, s_y	Source fragment origin	C	Colorimetric contraction
d_x, d_y	Destination fragment origin	Op	Colorimetric operation
T	Orthonormal transformation		
S	Size/shape of the region		

Table 1. Elements of a Fractal Code

Features from Fractals. The fractal representation of an image is an unordered set of fractal codes, which compactly describe the geometric alteration and colorization of fragments of a source image that will collage to form a destination image. Each fractal code yields a small set of features, formed by constructing subsets of its underlying tuple. These features thus afford position-, affine-, and colorimetric-agnosticism, as well as specificity.

Mutuality. The analogical relationship between two images may be seen as mutual; that is, image A is to image B as image B is to image A. However, the fractal representation is decidedly one-way (e.g. from A to B). To capture the bidirectional, mutual nature of the analogy between source and destination, we introduce the notion of a mutual fractal representation. Let us label the representation of the fractal transformation from image A to image B as T_{AB}. Correspondingly, we would label the inverse representation as T_{BA}. We shall define the mutual analogical relationship between A and B by the symbol M_{AB}, given by equation 1:

$$M_{AB} = T_{AB} \cup T_{BA} \tag{1}$$

By exploiting the set-theoretic nature of fractal representations T_{AB} and T_{BA} to express M_{AB} as a union, we afford the mutual analogical representation the complete expressivity and utility of the fractal representation. Further, the mutual fractal representation of the pairings may be extended to determine mutual fractal representations of triplets (equation 2) or quadruplets (equation 3) of images:

$$M_{ijk} = M_{ij} \cup M_{jk} \cup M_{ik} \tag{2}$$

$$M_{ijkl} = M_{ijk} \cup M_{ikl} \cup M_{jkl} \cup M_{ijl} \tag{3}$$

Therefore, in a mutual fractal representation, we have the apparatus necessary for reasoning analogically about the relationships between images, dependent upon only features which describe the mutual visual similarity present in those images.

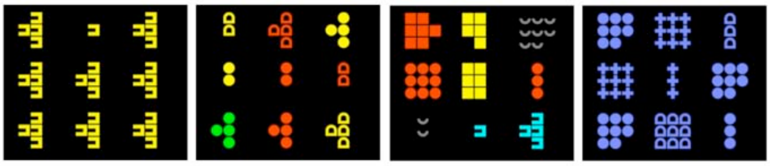

Fig. 1. Representative Odd One Out problems.

2.2 Odd One Out Problems

General one-one-out tasks can be presented with many kinds of stimuli, from words, colors, and images, to sets of objects. Minimal versions of these tasks are presented with three items, from which the "odd" one must be selected. Three item one-one-out tasks, in contrast to two-item response tasks, evaluate a participant's ability to compare relationships among stimuli, as opposed to just comparing stimuli features. It has been shown that these relationship-comparison tasks track general IQ measure more closely than do two-item tasks, and this tracking of IQ increases with the number of relationships to be considered [9]. We have chosen the Odd One Out test developed by Hampshire and colleagues at Cambridge Brain Sciences [11], which consists of matrix reasoning problems of varying levels of difficulty, in which the task is to decide which of the figures in the matrix does not belong.

Finding the Odd One Out, Fractally. Our technique for tackling the Odd One Out problems consists of three phases: segmentation, representation, and reasoning. First, we segment the problem image into nine subimages, I_1 through I_9. In the present implementation, the problems are given as 478x405 RGB-pixel JPEG images, with the subimages arrayed in a 3x3 matrix. At this resolution, each subimage fits well within a 96x96 pixel image.

Given the nine subimages, we group subimages into pairs, such that each subimage is paired once with the other eight subimages, forming 36 distinct pairings. We then calculate the mutual fractal representation M_{ij} for each pair of subimages I_i and I_j. The block partitioning used initially is identical to the largest possible block size, but subsequent recalculation of M_{ij} may be necessary using finer block partitioning. To determine the Odd One Out solely from the mutual fractal representations, we start by considering groupings of representations, beginning with pairings, and, if necessary, advance to consider other groupings.

Reconciling Multiple Analogical Relationships. For a chosen set of groupings G, we must determine how similar each member is to each of its fellow members. We first derive the features present in each member, as described above, and then calculate a measure of similarity as a comparison of the number of fractal features shared between each pair member [29].

We use the ratio model of similarity as described in [29], wherein the measure of similarity S between two representations A and B is calculated:

$$S(A,B) = f(A \cap B) / [f(A \cap B) + \alpha f(A-B) + \beta f(B-A)] \quad (4)$$

where f(X) is the number of features in the set X. To favor features from either image equally, we have chosen to set $\alpha = \beta = 1$ (the Jaccard similarity).

Relationship Space. As we perform this calculation for each pair A and B taken from the grouping G, we determine a set of similarity values for each member of G. We consider the similarity of each analogical relationship as a value upon an axis in a large "relationship space" whose dimensionality is determined by the size of the grouping. To arrive at a scalar similarity score for each member of the group G, we construct a vector in this multidimensional relationship space and determine its length, using the Euclidean distance formula. The longer the vector, the more similar two members are. As the Odd One Out problem seeks to determine, literally, "the odd one out," we seek to find the shortest vector, as an indicator of dissimilarity.

Distribution of Similarity. From the similarity score for a member of G, we determine subimage scoring by distributing the similarity value equally among the participating subimages. For each of the nine subimages, a score is generated which is proportional to its participation in the grouping. If a subimage is one of the two images in a pairing, as an example, then the subimage's similarity score receives one half of the pairing's calculated similarity score. Once all similarity scores of the grouping have been distributed to the subimages, the similarity score for each subimage is known. Although identifying which one among the subimages has the lowest similarity score, this may not yet sufficient for solving the problem, as ambiguity may be present.

Ambiguity. Similarity scores may vary widely. If the score for any subimage is unambiguously smaller than that of any other subimage, then the subimage is deemed "the odd one out." By unambiguous, we mean that there is no more than one score which is less than some ε, which we may vary as a tuning mechanism for the algorithm, and which we see as a useful yet coarse approximation of the boundary between the similar and the dissimilar in feature space. In practice, we calculate the deviation of each similarity measure from the average of all such measures, and use confidence intervals as a means for indicating ambiguity.

Refinement Strategy. However, if the scoring is inconclusive, then there are two readily available mechanisms at the algorithm's disposal: to modify the grouping such that larger sets of subimages are considered simultaneously (from pairs to triplets, or from triplets to quadruplets), or to recalculate the fractal representations using a finer partitioning. In our present implementation, we attempt bumping up the elements considered simultaneously as a first measure. If after reaching a grouping based upon quadruplets the scoring remains inconclusive, then we consider that the initial representation level was too coarse, and rerun the algorithm using ever finer partitions for the mutual fractal representation. If, after altering our considerations of groupings and examining the images at the finest level of resolution the scores prove inconclusive, the algorithm selects the subimage with the lowest score.

2.3 Analysis and Discussion

We have run our algorithm against 2,976 problems of the Odd One Out. These problems span a range of difficulty from the very easiest (level one) up to the most difficult (level 20). The performance ranged from nearly perfect on the easiest levels, to 70% correct at the middle difficulties, with a rapid falloff to 20% at the most difficult. For each problem, the choice of partitioning resolution was made automatically.

We note that most errors occur when the algorithm stops at quite high levels of partitioning. We interpret this as evidence that there exist levels-of-detail which are too gross to allow for certainty in reasoning. Indeed, the data upon which decisions are made at these levels are three orders of magnitude less than that which the finest partitioning affords. We find an opportunity for a refinement of the algorithm to assess its certainty based upon a naturally emergent artifact of the representation.

The errors that occurred at the finest level of partitioning are caused not due to the algorithm reaching an incorrect unambiguous answer but rather that the algorithm was unable to reach a sufficiently convincing or unambiguous answer. As we noted, these results are based upon calculations involving considering shifts in partitioning only, using pair wise comparisons of subimages. There appear to be Odd One Out problems for which considering pairs of subimages shall prove inconclusive at all available levels of detail. It is this set of problems which we believe implies that a shift in grouping (from pairs to triplets, or from triplets to quadruplets) must be undertaken to reach an unambiguous answer.

3 Fractal Perception and Action

In order to demonstrate that fractal analogies may form the basis of a theory of general intelligence, we need to describe how they can address the $f: P^* \rightarrow A$ mapping. To illustrate this we will construct an intelligent agent that lives in a simple simulated world similar to Reynolds's [26-27] boid worlds.

3.1 The Boid World

Schools of fish, murmurations of starlings, and stampedes of wildebeest are at once stunning and remarkable in appearance. The collection of agents, taken together, appear to be acting as if they were under some organized control.

Reynolds' boids are agents with an internal state which describes their current heading and an awareness of those agents to whom they should. They also have a minimum set of intrinsic behaviors that drive them to coordinate their actions with those flock mates: stay close together, don't collide, and mimic the motion of others.

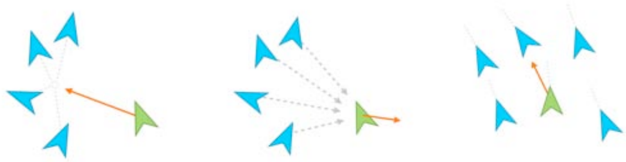

Fig. 2. Flocking Behaviors: Cohesion, Separation, and Alignment

Perception. A flock in nature may be composed of many thousands of individuals. It would seem an improbable computational load to place upon each agent within the flock the attempt to ascertain aspects of every member of the flock prior to making modifications to its own behavior. Some restriction of which individuals to consider must occur. Reynolds characterizes this as considering each agent to have a local perception. In computer simulations of flocks, the local perception each agent has of the world typically is provided to the agent by a godlike view of the entire environment, and a superimposed restriction of individuals by culling those deemed too distant to consider. This distance is usually referred to as a range of influence.

3.2 The Froid (Fractal Boid) World

For explorations of visual reasoning, affording agents with models of perception based on familiarity and novelty and observing those agents as flocks seems ideal. In our system, we endow our agents with a visual reasoning apparatus with the ability to receive the environment by localized observation only, and to perceive this received world via manipulations of fractal representations.

Froids versus Boids. Our agents, froids, sense and then classify their environment, whereas boids are told explicitly about their surrounds. Both boids and froids manifest the same behaviors, and thus participate in flocking with their mates, but only froids perceive and reason about their environment prior to enacting those behaviors. We establish a visual reasoning pipeline for a froid, from the reception of the world, through perceiving individuals and objects in the world, to reasoning about those perceptions, and finally, to enacting some course of action.

We made two simplifying architectural decisions for our experiment. First, the perception stage occurs in a serial fashion with the behavior decision stage, since the world of the simulation will not have changed until all the agents have moved themselves. Second, the perception stage would act only upon newly arriving stimuli, and not be influenced by prior decisions. We make these simplifications so that we may better compare the effect of perception on the subsequent behavior, without having our analysis take into account any perceptual hysteresis or other internal state.

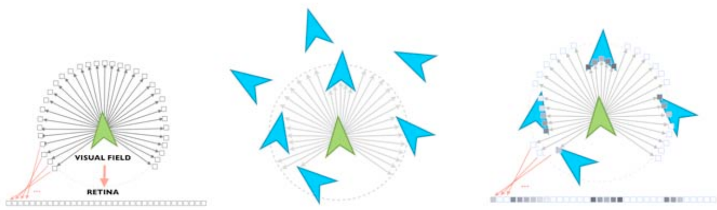

Fig. 3. Visual field to retina mapping, seeing via ray casting, and retinal objects

How a Froid Sees. We image a froid as having a single "eye" with a broad field of view. The froid's eye consists of a simulated retina, an arrangement of sensors. A froid sees its environment by receiving photometric stimulation upon this retina. The light entering each of these sensors is combined to form a visual field, as shown in figure 3-left. In our simulation, we use ray-casting to send a ray out through each of the sensors into the simulated world, and note whether that ray intersects anything. We illustrate this in Figure 3-center.

We interpret the "light" falling upon the sensor is a function of the distance of the intersected object from the froid, where objects which are distant are fainter than close objects. Figure 3-right shows an example of how objects within the froid's immediate environment may be mapped by this visual system onto its retina.

Fractal Perception. The photometric values arriving via the froid's retina next are interpreted by the froid's perception stage. For our present implementation we restrict the intentionality of the perception to only those tasks which will drive the flocking behavior. Accordingly, the primary task of the perception system is to determine flock mates.

This, however, raises an immediate question: what does a flock mate look like to a froid? Our froids are rendered into the simulated environment as chevrons whose orientation, color and physical size may vary. The visual environment, as transduced onto the retinal image, will show only an arranged set of values, roughly corresponding to visual distance to whatever object happened to intersect the ray from the sensor.

Filial Imprinting. There are many possible visual arrangements between a froid and a prototypical "other" in its environment. We chose to restrict our prototypes to six, four corresponding to points on the compass (north, south, east and west), and two corresponding to specific situations which would seem useful for behavior selection (close and empty). We refer to these as filial imprints, and they, along with their corresponding retinal impressions, are encoded into a fractal representation, and placed, indexed by derived fractal features, into the froid's memory system.

Finding the familiar by visual analogy. The arriving retinal image is an otherwise undifferentiated collection of photometric information, with each value corresponding to a particular direction and distance. From this retinal image, flock mates that might be within the visual range of the froid may be identified.

We begin by segmenting the retinal image into varying sets (collections of adjacent sensors), and then encoding each of these segments into fractal representations. We note that no attempt is made to interpret the retina image for edges or other boundary conditions: the segments are treated merely as they are found.

To determine the prototype P' which is most analogous to the retinal segment R from a set of fractal prototypes $P \coloneqq \{ P_1, P_2, \ldots P_n \}$:

$F \leftarrow$ Fractal(R, R)
Set $M \leftarrow 0$ and $P' \leftarrow$ unknown
For each prototype $P_i \in P$:
- Calculate the similarity of F to P_i : $S \leftarrow$ Sim(F, P_i)
- If $S > M$, then $M \leftarrow S$ and $P' \leftarrow P_i$

P' is therefore that prototype $P_i \in P$ which corresponds to the maximal similarity S, and is deemed the most analogous to retinal segment R.

Algorithm 1. Selecting the fractal familiar

If a segment corresponds to an imprinted prototype then we may make several inferences. The first is that an individual flock mate exists in that direction of view, which corresponds to the segment's retinal constituents. Secondly, we may infer that the flock mate lies at a distance which corresponds to a function of the faintness of the photometric readings of the retinal image. By systematically examining each segment of the retina, the froid's flock mates may be inferred by visual analogy.

3.3 The Three Laws for Froids

Once the flock mates have been discovered, the Reynolds rules for flocking may be invoked. Since the perception system has inferred the existence of a flock mate at a particular distance and direction, the **separation** and **cohesion** rules may be enacted directly. To **align** with a flock mate, the froid must infer the heading from the visual classification of the mate. This classification depends explicitly upon which of the filial prototypes has been selected as most representative of the retinal segment. We identified five rules of heading inference. Once the heading is inferred, the alignment rule of Reynolds may be used to adjust the motion of the froid.

3.4 Froids and Boids

To test our belief that a froid could behave as naturally as its boid counterparts, we created a traditional Reynolds boid system. We first placed into the environment several thousand standard boids, and observed that their aggregate motion was as expected: a realistic simulation of natural flocking behavior.

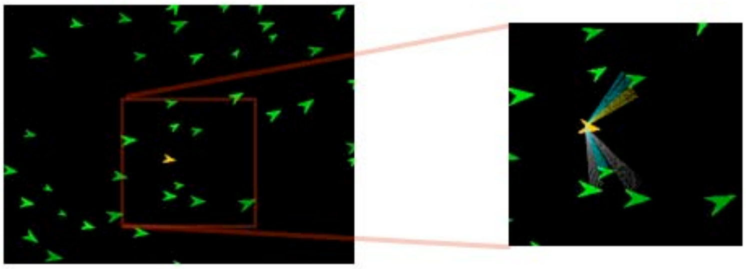

Fig. 4. A froid flocks with boids, and a closeup of the froid perceiving its environment

We then introduced one froid into the environment with the boids. Figure 4 shows a view of this simulation, with traditional boids in green, and the froid in gold. We observed that the froid behaved in the same manner as those boids whose identification of flock mates was given in the traditional oracle manner.

We note that, unlike the boids, the froids appeared to suffer from uncertainty (manifested by a stuttering motion) when in the proximity of a large number of other boids. We surmised that this is due to the inability of the segmentation system using within the retina to accommodate or otherwise classify large amounts of overlapping or confounding visual data. Another possibility concerns the enaction itself. Let us suppose that two action vectors arising due to two received perceptual signals almost exactly cancel each other. In this case, small fluctuations in the perceptual signal can cause a significant change in the action vector, which may result in stuttering.

4 Conclusion

In earlier work [5-6], we showed that visual knowledge and reasoning alone could address some classes of analogy problems that had been assumed to require causal knowledge and reasoning. We also showed how visual analogies could account for several aspects of creative problem solving in scientific discovery [8] and engineering design [7]. However, this work still used propositional representations, while the content of knowledge was visuospatial. In [16-17], we showed how visual knowledge represented iconically can address analogy problems on the Raven's Progressive Matrices test of intelligence. Previously, the visual analogy problems on the Raven's test had been assumed to require propositional representations. The Raven's test also formed the context of our first development of fractal representations for addressing visual analogy problems [22]. The fractal method on the Raven's test performs about as well as typically human teenager. Hertzmann et al [13] have used a different fractal technique for comparing texture in two images.

In this paper, first we showed that an improved fractal technique can address visual analogy problems on the Odd One Out test of intelligence at the level of most adult humans. Further, the fractal technique imitates two important features of human performance: starting with low-level relationships and moving to higher relationships if and as needed, and automatic adjustment of the level of resolution to resolve ambiguities. We posit that fractal representations are knowledge representations in the sense of Biederman [3] in that they encode the relationship between non-accidental perceptual constructs within an image. We posit further that fractals are knowledge representations in the deep sense of Davis, Shrobe & Szolovits [4] in which representation and reasoning are closely intertwined.

Then, in this paper we that showed the fractal technique for visual analogies can be used for perception. We demonstrated that froids (fractal-based boids) can use the fractal technique for mapping percepts into actions which manifest flocking behavior. The froids used a simple architecture called "reactive control" in robotics [1] and "situated action" in cognitive science [23], directly mapping percepts into actions.

While the use of fractal representations is central to our technique, the emphasis upon visual recall in our solution afforded by features derived from those representations is also important. There is evidence that certain species have innate or rapidly develop through acclimation visual prototypes which allow young members to accurately identify their parents [25]. We hold that placing imprints into memory, indexed via fractal features, affords a new and robust method of discovering image similarity, and that images, encoded and represented in terms of themselves, may be indexed and retrieved without regard to shape, geometry, or symbol.

Our goal is to develop a Fractal Theory of General Intelligence. We believe that in this paper we have taken two important steps in that long journey: we have demonstrated that (1) our fractal technique can address visual analogy problems on intelligence tests on par with human performance, and (2) our fractal technique enables real-time percept-to-action mapping capable of imitating flocking behavior, at least in a simulated world.

5 References

1. Arkin, R. 1998. *Behavior-Based Robotics*. Boston, MA: The MIT Press.
2. Barnsley, M., and Hurd, L. 1992. Fractal Image Compression. Boston, MA: A.K. Peters.
3. Biederman, I. 1987. Recognition-by-Components: A Theory of Human Image Understanding. *Psychological Review,* 94, 115-147.
4. Davis, R. Shrobe H., and Szolovits, P. 1993. What is a Knowledge Representation? AI Magazine 14.1:17-33.
5. Davies, J., and Goel, A. 2001 Visual Analogy in Problem Solving. In *Proc. 17th International Joint Conference on Artificial Intelligence* (IJCAI-01), August 2001, pp. 377-382. Morgan Kaufmann.
6. Davies, J., Goel, A., & Yaner, P. 2008. Proteus: Visuospatial Analogy in Problem Solving. *Knowledge-Based Systems* 21(7):636-654, October 2008.
7. Davies, J., Goel, J., & Nersessian, N. 2009. A Computational Model of Visual Analogies in Design. *Cognitive Systems Research,* 10:204-215.
8. Davies, J., Nersessian, N., & Goel, A. 2005 Visual Models in Analogical Problem Solving. *Foundations of Science*, 10(1):133-152, 2005.
9. Diascro, M. N., & Brody, N. 1994. Odd-man-out and intelligence. *Intelligence,* 19(1), 79-92.
10. Gentner, D. 1983. Structure-Mapping: A Theoretical Framework for Analogy. *Cognitive Science* 7(2), 155-170.

11. Hampshire, A. 2010. The Odd One Out Test of Intelligence. <http://www.cambridgebrainsciences.com/browse/reasoning/test/oddoneout>
12. Haugeland, J. (editor, 1981) Mind Design: Philosophy, Psychology and Artificial Intelligence, MIT Press.
13. Hertzmann, A., Jacobs, C. E., Oliver, N., Curless, B., & Salesin, D. 2001. Image analogies, in *Computer Graphics*, 25(4) *(SIGGRAPH '01 Conference Proceedings)*: 327–340.
14. Hofstadter, D., & Fluid Analogies Research Group (Eds.). 1995. *Fluid concepts & creative analogies: Computer models of the fundamental mechanisms of thought*. New York: Basic Books.
15. Indurkhya, B. (1998). On creation of features and change of representation. Journal of Japanese Cognitive Science Society, 5(2), 43-56.
16. Kunda, M., McGreggor, K. and Goel, A. 2010. Taking a Look (Literally!) at the Raven's Intelligence Test: Two Visual Solution Strategies. In Proc. 32nd Annual Meeting of the Cognitive Science Society, Portland, August 2010.
17. Kunda, M., McGreggor, K. and Goel, A. In Press. A computational model for solving problems from the Raven's Progressive Matrices intelligence test using iconic visual representations. To appear in Cognitive Systems Research.
18. Lovett, A., Lockwood, K., & Forbus, K. 2008. Modeling cross-cultural performance on the visual oddity task. Proceedings of Spatial Cognition 2008.
19. Mandelbrot, B. 1982. The fractal geometry of nature. San Francisco: W.H. Freeman.
20. Markou, M., & Singh, S. 2003a. Novelty Detection: A Review- Part 1: Statistical Approaches. Signal Processing, 83(12): 2481-2497.
21. Markou, M., & Singh, S. 2003b. Novelty Detection: A Review- Part 2: Neural Network Based Approaches. Signal Processing, 83(12): 2481-2497.
22. McGreggor, K., Kunda, M., and Goel, A. 2011. Fractal Analogies: Preliminary Results from the Raven's Test of Intelligence. In Proc. International Conference on Computational Creativity, Mexico City, Mexico, April 27-29.
23. Norman, D. 1993. Cognition in the Head and in the World: An Introduction to the Special Issue on Situated Action. *Cognitive Science* 17:1-6.
24. Neto, H. & Nehmzow, U. 2007. Visual Novelty Detection with Automatic Scale Selection. Robotics and Autonomous Systems, 55(693-701).
25. O'Reilly, Randall C., and Johnson, Mark H., 1994. Object Recognition and Sensitive Periods: A Computational Analysis of Visual Imprinting. *Neural Computation* 6: 357-389.
26. Reynolds, C. W. 1987. Flocks, Herds, and Schools: A Distributed Behavioral Model, in *Computer Graphics*, 21(4) (*SIGGRAPH '87 Conference Proceedings*): 25-34.
27. Reynolds, C. W. 1999. Steering Behaviors For Autonomous Characters, in *Proceedings of Game Developers Conference 1999* held in San Jose, California. Miller Freeman Game Group, San Francisco, California. 763-782.
28. Russell, S., & Norvig, P. (2003) *Artificial Intelligence: A Modern Approach.* Prentice-Hall.
29. Tversky, A. 1977. Features of similarity. Psychological Review, 84(4), 327-352.
30. Wagemans, J., Elder, J., Kubovy, M., Palmer, S., Peterson, M., Singh, M., von der Heydt, R. In press A. A century of Gestalt psychology in visual perception: I. Perceptual grouping and figure-ground organization. *Psychological Bulletin*.
31. Wagemans, J., Feldman, J., Gerpshtein, S., Kimchi, R., Pomerantz, J., van der Helm, P. van Leeuwen, In press B. A Century of Gestalt Psychology in Visual Perception II. Conceptual and Theoretical Foundations. *Psychological Bulletin*.